DTU Library

Nonlocal incoherent solitons

Krolikowski, Wieslaw; Bang, Ole; Wyller, John

Published in:
Physical Review E

Link to article, DOI:
10.1103/PhysRevE.70.036617

Publication date:
2004

Document Version
Publisher's PDF, also known as Version of record

Citation (APA):
Krolikowski, W., Bang, O., & Wyller, J. (2004). Nonlocal incoherent solitons. *Physical Review E*, *70*(3), [036617]. https://doi.org/10.1103/PhysRevE.70.036617

PHYSICAL REVIEW E **70**, 036617 (2004)

Nonlocal incoherent solitons

Wiesław Królikowski
Laser Physics Centre, Research School of Physical Sciences and Engineering, The Australian National University, Canberra, Australian Capital Territory 0200, Australia

Ole Bang
Research Center COM, Technical University of Denmark, 2800 Kongens Lyngby, Denmark

John Wyller*
Nonlinear Physics Centre and Laser Physics Centre, Research School of Physical Sciences and Engineering, The Australian National University, Canberra, Australian Capital Territory 0200, Australia
(Received 13 May 2004; published 30 September 2004)

We investigate the propagation of partially coherent beams in spatially nonlocal nonlinear media with a logarithmic type of nonlinearity. We derive analytical formulas for the evolution of the beam parameters and conditions for the formation of nonlocal incoherent solitons.

DOI: 10.1103/PhysRevE.70.036617 PACS number(s): 42.65.Tg, 42.65.Jx, 42.25.Kb

Incoherent solitons are partially coherent optical beams propagating without changing their shape in materials with a slow nonlinear response [1,2]. Such nonlinear materials respond to the time averaged light intensity, which means that even though the phase of the partially coherent light beam fluctuates randomly, it still leads to a smooth light-induced refractive index change and subsequent trapping of the beam [3–9]. Incoherent solitons have been experimentally observed almost solely in photorefractive media, which exhibit a strong and sufficiently slow saturable Kerr-like nonlinearity [1,2].

In inertial bulk Kerr media $(2+1)$-dimensional partially coherent beams are still unstable and will either diffract or self-focus unboundedly, depending on whether the power is above a certain critical value [10–12]. Incoherence counteracts self-focusing and leads to an increase of the critical power, but it cannot remove the collapse instability [10,11]. Consequently all experimental observations of incoherent solitons have been in saturable materials, such as photorefractive materials [1] and liquid crystals [13].

Recently Peccanti *et al.* reported the first observation of incoherent solitons and their interaction in nematic liquid crystals [13]. Liquid crystals have a strong noninstantaneous saturable Kerr-like nonlinearity associated with light-induced molecular reorientation [14–17]. Their nonlinear response time ranges from tens to hundreds of milliseconds [14], which is sufficiently slow to allow for the formation of incoherent solitons. Apart from being inertial the nonlinear response of liquid crystals is also inherently *spatially nonlocal*, because the molecular reorientation induced by a light beam in a particular place will affect the orientation of molecules far beyond this point, due to the long-range character of their interaction [15,18].

Spatially nonlocal nonlinearities are in fact an inherent property of many physical systems, including matter waves, nonlinear optics, and field theory. As such the nonlocal aspect of nonlinear interaction has attracted significant interest recently [19,20]. Nonlocality can have profound consequences on the properties of optical beams and soliton formation, e.g., leading to collapse arrest of finite-size beams [21], attraction and formation of bound states of dark solitons [22], and modulational instability (MI) in defocusing materials [23,24]. A nonlocal nonlinearity may even describe parametric wave mixing and solitons [25]. In the particular case of liquid crystals certain aspects of their nonlocal nonlinear response have already been discussed in the literature, including theoretical and experimental studies of the formation of multiple solitons [26,27] and accessible solitons [28] as well as soliton interaction and MI [29].

Motivated by the documented unique dual inertial and nonlocal character of the nonlinearity of liquid crystals and the recent experimental observation of incoherent solitons in this material, we analyze here the effect of nonlocality on the propagation of partially coherent beams and formation of incoherent solitons. Because such a problem is in general analytically intractable we consider here the special case of a nonlinearity with a logarithmic dependence of the refractive index change on light intensity. This model has previously been used to study incoherent beams in local media [4,6,30–32] as well as fully coherent beams in nonlocal media [33]. Here we extend it to describe *simultaneously incoherence and nonlocality*. While being rather specific, the logarithmic model is unique in the extent that it enables a fully analytical treatment of the nonlinear beam propagation. It has been successfully used to predict a variety of effects associated with beam propagation and soliton formation [31,34].

Propagation of a two-dimensional quasimonochromatic partially coherent beam with the slowly varying amplitude $\psi(\vec{r},z)$ is governed by the following nonlinear Schrödinger equation:

*On sabbatical leave from the Department of Mathematical Sciences and Technology, Agricultural University of Norway, P. O. Box 5003, N-1432 Ås, Norway.

$$i\frac{\partial \psi}{\partial z} + \frac{1}{2}\nabla_{\vec{r}}^2\psi + \delta n(I)\psi = 0, \tag{1}$$

where $\vec{r}=(x,y)$, $I=|\psi|^2$ denotes the light intensity, $\delta n(I)$ represents the light-induced refractive index change, and $\nabla_{\vec{r}}^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$.

In the following we use the mutual coherence function $\Gamma(\vec{r}_1,\vec{r}_2,z)$ to represent the coherence properties of the beam [35]. This function describes the spatial correlation between the field at points $\vec{r}_1$ and $\vec{r}_2$, and is defined as

$$\Gamma(\vec{r}_1,\vec{r}_2,z) = \langle \psi(\vec{r}_1,z)\psi^*(\vec{r}_2,z)\rangle, \tag{2}$$

where the angular brackets denote temporal or ensemble averaging. In terms of the mutual coherence function the (time averaged) intensity is given by

$$I(\vec{r},z) = \Gamma(\vec{r},\vec{r},z). \tag{3}$$

We take the refractive index change δn to be the following *nonlocal* function of light intensity:

$$\delta n(\vec{r},I) = n_2 \ln\left[\int R(\vec{r}-\vec{\xi})I(\vec{\xi})d\vec{\xi}\right], \tag{4}$$

where $\int d\vec{r} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}dxdy$ and $R(\vec{r})=R(r)$ is the so-called nonlocal response function, which depends only on the length of its argument vector $r=|\vec{r}|=\sqrt{x^2+y^2}$.

The model (4) is phenomenological and reflects the fact that the refractive index change in a particular point in space is determined not only by the wave intensity in this point, but also by the intensity in a certain surrounding region given by the width of the response function. The width of the nonlocal response function therefore determines the degree of nonlocality. In the special case when $R(\vec{r})=\delta(r)$ this model describes a local nonlinear medium $\delta n = n_2 \ln I$. A simplified model may also be derived in the weakly nonlocal limit, in which the width of the response function is finite, but small compared to the beam width [20,36]. In the strongly nonlocal limit, in which the response function is much broader than the beam, the governing model becomes linear [20,33].

In general, the particular form of the response function is determined by the specifics of the physical process responsible for the nonlinearity of the optical medium. For instance, it can be shown that for the reorientational nonlinearity of liquid crystals and general diffusion type nonlinearities, the nonlocal response can be approximated as

$$R(\vec{r}) \propto \exp(-r/\sigma), \tag{5}$$

with σ representing the extent of the nonlocality [25]. However, the majority of nonlocality-mediated effects appear to be rather generic [20,23] and do not depend strongly on the particular form of the nonlocal response function. Here, to enable us to make an analytical description, we therefore assume that the nonlocality is described by the normalized Gaussian response function

$$R(r) = \frac{1}{\pi\sigma^2}\exp\left(-\frac{x^2+y^2}{\sigma^2}\right). \tag{6}$$

While we consider here a nonlocally isotropic medium, the model can be easily extended to include anisotropy of the nonlocal properties, such as those caused by a directional heat flow in thermal media [37].

The mutual coherence function satisfies the following propagation equation [11,12]

$$i\frac{\partial \Gamma}{\partial z} + \frac{1}{2}(\nabla_{\vec{r}_1}^2 - \nabla_{\vec{r}_2}^2)\Gamma + [\delta n(\vec{r}_1,I) - \delta n(\vec{r}_2,I)]\Gamma = 0. \tag{7}$$

Introducing the independent spatial variables $\vec{p}$ and $\vec{q}$,

$$\vec{p} = \frac{1}{2}(\vec{r}_1+\vec{r}_2), \quad \vec{q} = \vec{r}_1 - \vec{r}_2, \tag{8}$$

and using the specific form (4) of the nonlinearity transforms the propagation equation into

$$i\frac{\partial \Gamma}{\partial z} + \nabla_{\vec{p}}\cdot\nabla_{\vec{q}}\Gamma + n_2 \ln\left(\frac{\int R(\vec{r}_1-\vec{\xi})I(\vec{\xi})d\vec{\xi}}{\int R(\vec{r}_2-\vec{\xi})I(\vec{\xi})d\vec{\xi}}\right)\Gamma = 0. \tag{9}$$

In what follows we will consider the so-called Gaussian-Schell model for the incident partially coherent beam [35]. We will also restrict our analysis to circular beams, i.e., beams having isotropic coherence properties. The extension of the analysis to elliptical incoherent beams is straightforward. The mutual coherence function of the Gaussian-Schell beam is expressed as

$$\Gamma(\vec{p},\vec{q},z=0) = \exp\left(-\frac{|\vec{p}|^2}{\rho_0^2} - \frac{|\vec{q}|^2}{\theta_0^2}\right), \tag{10}$$

where the initial effective coherence radius θ_0 is given by the relation

$$1/\theta_0^2 = 1/r_c^2 + 1/(4\rho_0^2). \tag{11}$$

Here ρ_0 is the initial radius and r_c is the initial coherence radius of the beam, respectively. Due to the logarithmic form of the nonlinearity, the beam will maintain the Gaussian statistics during propagation and thus the modulus of the coherence function will keep the form given by Eq. (10). We therefore look for the solutions to Eq. (9) using the following Gaussian ansatz:

$$\Gamma(\vec{p},\vec{q},z) = A(z)\exp\left(-\frac{|\vec{p}|^2}{\rho^2(z)} - \frac{|\vec{q}|^2}{\theta^2(z)} + i\vec{p}\cdot\vec{q}\mu(z)\right), \tag{12}$$

where $A(z)$ and $\mu(z)$ represent the amplitude and phase variations of the coherence function, and $\rho(z)$ and $\theta(z)$ are its radius and generalized coherence radius, respectively. The input amplitude plays no role for the dynamics and thus we use the initial conditions $A(0)=1$, $\rho(0)=\rho_0$, and $\theta(0)=\theta_0$. In addition, we assume $\mu(z=0)=0$ so the coherence function $\Gamma(\vec{p},\vec{q},z)$ satisfies the initial condition (10) at $z=0$. Inserting the ansatz (12) into Eq. (9) leads to a set of ordinary differential equations for the parameters of the coherence function

$$\frac{d\theta}{dz} = \theta\mu, \tag{13}$$

$$\frac{d\rho}{dz} = \rho\mu, \tag{14}$$

$$\frac{dA}{dz} = -2A\mu, \tag{15}$$

$$\frac{d\mu}{dz} = \frac{4}{\theta^2\rho^2} - \mu^2 - \frac{2n_2}{\rho^2+\sigma^2}. \tag{16}$$

From the first two equations (13) and (14) we obtain the relation

$$\theta/\rho = \theta_0/\rho_0, \tag{17}$$

which shows that during the evolution the beam conserves its coherence, defined as the number of speckles within the beam diameter. Equations (14) and (15) lead to the power conservation relation $A\rho(z)^2=\rho_0^2$. Interestingly, neither relation depends on whether or not the nonlinearity is local.

Combining Eq. (14), (16), and (17) we obtain the following evolution equation for the beam radius:

$$\frac{d^2\rho}{dz^2} - 4\frac{\rho_0^2}{\rho^3\theta_0^2} + 2n_2\frac{\rho}{\rho^2+\sigma^2} = 0. \tag{18}$$

This equation describes the dynamics of the width of a partially coherent beam with Gaussian statistics in a nonlocal medium with a logarithmic nonlinear response. Taking $\sigma=0$ we recover the expressions governing incoherent beams in a local medium [31]. On the other hand, for $r_c \to \infty$, Eq. (18) describes coherent beams in a nonlocal medium. It is clear that the main role of the nonlocality is to effectively weaken the influence of the nonlinearity on the propagation of the beam. For very high degrees of nonlocality the nonlinear effects become negligible and the beam just diffracts.

Incoherent nonlocal soliton solutions are obtained from Eq. (18) by setting $\rho(z)=\rho_0$. This gives the relation

$$n_2 = \left(\frac{2}{r_c^2} + \frac{1}{2\rho_0^2}\right)\left(1 + \frac{\sigma^2}{\rho_0^2}\right). \tag{19}$$

As expected, the existence of bright incoherent solitons, or localized beams, is seen to require a focusing nonlocal nonlinearity with $n_2>0$. Importantly, the relation (19) further shows that trapping of a beam with a given initial width ρ_0 and coherence radius r_c in a nonlocal medium requires a *stronger nonlinearity* (higher n_2) than in the case of a purely local nonlinear response. This is again due to the fact that the nonlocality effectively leads to a decrease of the strength of focusing and subsequently to a weaker localization of the beam. From Eq. (19), we find the expression for the radius of the incoherent nonlocal soliton

$$\rho_0^2 = \frac{\tilde{\rho}_0^2}{2}\left[1 + \frac{4\sigma^2}{r_c^2} + \sqrt{\left(1+\frac{4\sigma^2}{r_c^2}\right)^2 + \frac{4\sigma^2}{\tilde{\rho}_0^2}}\right], \tag{20}$$

where

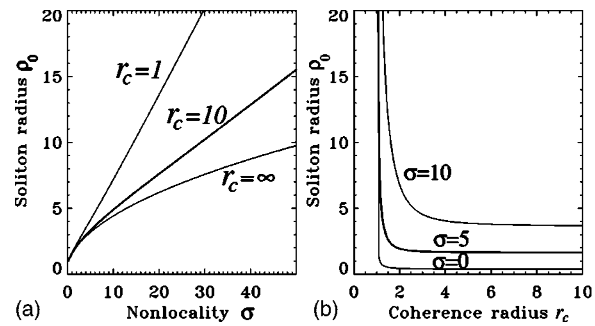

FIG. 1. Soliton width (ρ) as a function of the degree of nonlocality (a) and coherence radius (b). Here $n_2=1.8$.

$$\tilde{\rho}_0^2 \equiv \rho_0(\sigma=0) = \frac{1}{2n_2 - 4/r_c^2} \tag{21}$$

denotes the radius of the incoherent soliton in a local medium.

In Fig. 1(a) we plot the radius of incoherent nonlocal solitons versus the degree of nonlocality σ for three values of the coherence radius r_c. These plots clearly show that the soliton width increases with nonlocality and becomes proportional to σ in the highly nonlocal regime. One can show that for $\sigma/r_c \gg 1$

$$\rho_0 = 2\tilde{\rho}_0\frac{\sigma}{r_c}. \tag{22}$$

The degree of nonlocality is an inherent property of the medium and as such difficult to control. However, one can easily vary the coherence properties of an incident beam. Therefore we plot in Fig. 1(b) the soliton width ρ_0 versus the coherence radius r_c for three values of the nonlocality σ. Again, the increase of the soliton width with incoherence and nonlocality is evident. Additionally, Fig. 1(b) shows that for a given strength of nonlinearity there is a minimum coherence radius, below which the incoherent soliton cannot exist. The existence of such a threshold is well known from earlier works on incoherent solitons in local media [4,6,31]. Interestingly, it turns out that for a nonlocal nonlinearity, this threshold is also determined solely by the strength of the nonlinearity and the degree of coherence through the relation

$$n_2 - 2/r_c^2 > 0. \tag{23}$$

To understand why this condition does not depend on the beam size or the degree of nonlocality we recall that the soliton is formed when the nonlinearity fully compensates the transverse spreading of the beam, which is caused by incoherence and diffraction. The former is due to the diffusive character of the beam, while the latter is a direct consequence of the finite size of the beam. For a very broad beam the contribution to its expansion due to diffraction and nonlocality becomes negligible compared to that due to incoherence. Hence, to achieve beam trapping the strength of the nonlinear response of the medium should be high enough to compensate at least the incoherence-induced beam spreading. This, in fact, determines the threshold for soliton generation. As soon as this condition is satisfied a bright inco-

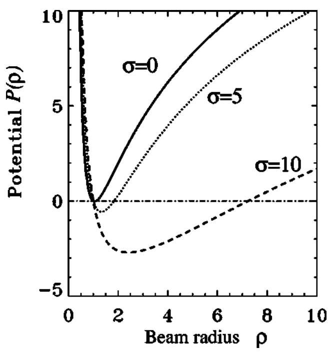

FIG. 2. Potential $P(\rho)$ in a focusing logarithmic nonlinear medium ($n_2=1.8$) for different degrees of nonlocality σ. The initial beam parameters are $\theta_0=1$ and $\rho_0=1$.

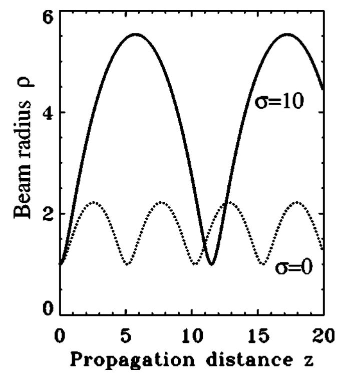

FIG. 3. Illustration of nonstationary propagation of a partially coherent beam in a logarithmic nonlinear nonlocal medium. The nonlocality parameter varies as $\sigma=0$ (dotted line) and $\sigma=10$ (solid line).

herent soliton can exist, whose width will be determined by the interplay of nonlinearity, coherence, diffraction, and nonlocality. Obviously, the resulting beam diameter will always be larger than that of the local soliton of the same degree of coherence.

More physical insight into the competition between diffraction, nonlinearity, nonlocality, and incoherence can be obtained by an effective particle analogy. Assuming that $(d\rho/dz)(z=0)=0$, and integrating Eq. (18) once, one obtains a classical-mechanical equation

$$(d\rho/dz)^2 + P(\rho) = 0, \tag{24}$$

describing an effective particle ρ with kinetic energy $(d\rho/dz)^2$ moving in the potential $P(\rho)$, given by

$$P(\rho) = \frac{4}{\theta_0^2}\left(\frac{\rho_0^2}{\rho^2} - 1\right) + 2n_2 \ln\left(\frac{\sigma^2+\rho^2}{\sigma^2+\rho_0^2}\right). \tag{25}$$

The asymmetric potential is depicted in Fig. 2 for a beam with effective coherence radius $\theta_0=1$ moving in a focusing logarithmic nonlinear medium with $n_2=1.8$. As nonlocality increases the width of the potential also increases and its minimum becomes less pronounced.

Stationary soliton solutions with constant beamwidth correspond to the effective particle being located at the bottom of the potential well. For initial beam parameters different from the soliton solution the beam width (and coherence radius and peak intensity) will undergo periodic oscillations, corresponding to the effective particle oscillating in the bottom of the potential well. As the profile of the potential well changes with higher σ one can expect that the amplitude and period of beam oscillations will drastically increase for a high degree of nonlocality. In the limit of the small amplitude oscillations, their form can be obtained analytically in the form of Jacobi elliptical function, in a way analogous to that discussed in [31].

This is indeed the case, as illustrated in Fig. 3 where we show the nonstationary propagation of the partially coherent beam in a nonlinear medium. The initial parameters are chosen such that $\rho_0=1.0$, $r_c=1.15$, and $n_2=1$. In the graph we plot the beam radius as a function of the propagation distance in the cases of local (dotted line) and nonlocal (solid line) nonlinearity. The increased amplitude and period of oscillations induced by the nonlocality is evident.

In conclusion, we analyzed the propagation of incoherent optical beams with Gaussian statistics in a nonlinear nonlocal medium. Considering a special case of a logarithmic type of nonlinearity and Gaussian nonlocal response, we obtained analytical formulas governing all relevant parameters of the beam. We showed that nonlocality results in increased diameter of the incoherent solitons. When the initial parameters of the beam differ from the exact nonlocal soliton solution, the beam experiences periodic expansions and contractions whose period increases with nonlocality.

W.K. was supported by the Australian Research Council. J.W. acknowledges support from The Research Council of Norway under Grant No. 153405/432.

[1] M. Mitchell, Z. Chen, M. Shih, and M. Segev, Phys. Rev. Lett. **77**, 490 (1996).
[2] M. Mitchell and M. Segev, Nature (London) **387**, 880 (1997).
[3] D. N. Christodoulides, T. H. Coskun, M. Mitchell, and M. Segev, Phys. Rev. Lett. **78**, 646 (1997).
[4] D. N. Christodoulides, T. H. Coskun, and R. I. Joseph, Opt. Lett. **22**, 1080 (1997).
[5] M. Mitchell, M. Segev, T. H. Coskun, and D. N. Christodoulides, Phys. Rev. Lett. **79**, 4990 (1997).
[6] D. N. Christodoulides, T. H. Coskun, M. Mitchell, and M. Segev, Phys. Rev. Lett. **80**, 2310 (1998).
[7] M. I. Carvalho, T. H. Coskun, D. N. Christodoulides, M. Mitchell, and M. Segev, Phys. Rev. E **59**, 1193 (1999).
[8] A. W. Snyder and D. J. Mitchell, Phys. Rev. Lett. **80**, 1422 (1998).
[9] V. V. Shkunov and D. Z. Anderson, Phys. Rev. Lett. **81**, 2683 (1998).
[10] O. Bang, D. Edmundson, and W. Krolikowski, Phys. Rev. Lett.

83, 5479 (1999).

[11] V. A. Aleshkevich, S. S. Lebedev, and A. N. Matveev, Sov. J. Quantum Electron. **11**, 647 (1981).

[12] G. A. Pasmanik, Sov. Phys. JETP **39**, 243 (1974).

[13] M. Peccianti and G. Assanto, Opt. Lett. **26**, 1791 (2001).

[14] M. Peccianti, K. A. Brzdakiewicz, and G. Assanto, Opt. Lett. **27**, 1460 (2002).

[15] G. Assanto and M. Peccianti, IEEE J. Quantum Electron. **39**, 13 (2003).

[16] M. A. Karpierz, M. Sierakowski, M. Swillo, and T. Wolinski, Mol. Cryst. Liq. Cryst. Sci. Technol., Sect. A **320**, 157 (1998).

[17] J. Beeckman, K. Neyts, X. Hutsebaut, C. Cambournac, and M. Haelterman, Opt. Express **12**, 1011 (2004).

[18] X. Hutsebaut, C. Cambournac, M. Haelterman, J. Beeckman, and K. Neyts (unpublished).

[19] W. Krolikowski, O. Bang, J. Wyller, and J. J. Rasmussen, Acta Phys. Pol. A **103**, 133 (2003).

[20] W. Krolikowski, O. Bang, N. Nikolov, D. Neshev, J. Wyller, J. J. Rasmussen, and D. Edmundson, J. Opt. B: Quantum Semiclassical Opt. **6**, S288 (2004).

[21] O. Bang, W. Krolikowski, J. Wyller, and J. J. Rasmussen, Phys. Rev. E **66**, 046619 (2002).

[22] N. I. Nikolov, D. Neshev, W. Krolikowski, O. Bang, J. J. Rasmussen, and P. L. Christiansen, Opt. Lett. **29**, 286 (2004).

[23] J. Wyller, W. Krolikowski, O. Bang, and J. J. Rasmussen, Phys. Rev. E **66**, 066615 (2002).

[24] W. Krolikowski, O. Bang, J. J. Rasmussen, and J. Wyller, Phys. Rev. E **64**, 016612 (2001).

[25] N. I. Nikolov, D. Neshev, O. Bang, and W. Krolikowski, Phys. Rev. E **68**, 036614 (2003).

[26] X. Hutsebaut, C. Cambournac, M. Haelterman, A. Adamski, and K. Neyts, Opt. Commun. **233**, 211 (2004).

[27] D. W. McLaughlin, D. J. Muraki, and M. J. Shelley, Physica D **97**, 471 (1996).

[28] C. Conti, M. Peccianti, and G. Assanto, Phys. Rev. Lett. **91**, 073901 (2003); **92**, 113902 (2004).

[29] M. Peccianti, C. Conti, and G. Assanto, Phys. Rev. E **68**, 025602 (2003).

[30] A. Snyder and J. Mitchell, Opt. Lett. **22**, 16 (1997).

[31] W. Królikowski, D. Edmundson, and O. Bang, Phys. Rev. E **61**, 3122 (2000).

[32] S. A. Ponomarenko, Phys. Rev. E **64**, 036618 (2001); S. A. Ponomarenko and G. P. Agrawal, *ibid.* **69**, 036604 (2004).

[33] A. Snyder and J. Mitchell, Science **276**, 1538 (1997); J. Opt. Soc. Am. B **16**, 236 (1999).

[34] I. Bialynicki-Birula and J. Mycielski, Phys. Scr. **20**, 539 (1979).

[35] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, Cambridge, England, 1995).

[36] W. Krolikowski and O. Bang, Phys. Rev. E **63**, 016610 (2001).

[37] M. Bertolotti, R. Li Voti, S. Marchetti, and C. Sibilia, Opt. Commun. **133**, 578 (1997).